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Sem 1 Examination 2010
Question/answer booklet

MATHEMATICS: SPECIALIST
3CDMAS
Section One
(calculator-free)

Name: ______________________________

Teacher: ______________________________

Time allowed for this section
Section One
Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes

Material required/recommended for this paper
To be provided by the supervisor
Question/answer booklet for Section One and a formula sheet (from Curriculum Council) which can be used for Section Two.

To be provided by the candidate
Section One:
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler

Important note to candidates
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Structure of this paper

Section	Number of questions available	Number of questions to be answered	Suggested working time (minutes)	Marks available
Section One: Calculator-free	**7**	**7**	**50**	**39**
Section Two: Calculator-assumed	13	13	100	79
				118

Instructions to candidates

1. The rules for the conduct of Western Australian examinations are detailed in the *Year 12 Information Handbook 2010*. Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions according to the following instructions.
   Section One: Write answers in this Question/Answer Booklet. **All** questions should be answered.
   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
   It is recommended that you **do not use pencil** except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

MARK ALLOCATION AND RECORDS:

Section	Question	Marks	Awarded
ONE	1	3	
	2	3	
	3	4	
	4	6	
	5	6	
	6	7	
	7	10	
	Penalties	- 1/2/3	
	ONE	**39**	
	TWO	**79**	
	TOTAL	**118**	

Penalties	
Rounding (-1)	
Units (-1)	
Notation (-1)	

%

Section One (calculator-free) 39 marks

This section has **SEVEN (7)** questions. Attempt **all** questions.

Working time: **50 minutes**

The following exact value table may be useful to answer questions in this examination.

	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

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1. [3 marks]

Find the value(s) of a if $\langle 6, a, -4\rangle$ and $\langle -1, 2a, a\rangle$ are perpendicular.

$\langle 6, a, -4\rangle \circ \langle -1, 2a, a\rangle = 0$

$-6 + 2a^2 - 4a = 0$ ✓

$a^2 - 2a - 3 = 0$

$(a-3)(a+1) = 0$

$\therefore a = 3$ ✓ or -1 ✓

2. [3 marks]

Find $\frac{dy}{dx}$ for $x^2 + 2xy - y^2 = 7$

$\frac{d}{dx}(x^2) + 2\frac{d}{dx}(xy) - \frac{d}{dx}(y^2) = \frac{d}{dx}(7)$

$2x + 2y + 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$ ✓

$x + y = \frac{dy}{dx}(y - x)$ ✓

$\frac{dy}{dx} = \frac{x+y}{y-x}$ ✓

3. [4 marks]

Find $\frac{d}{dx}(x \sin x)$ and hence $\int x \cos x \, dx$.

$\frac{d(x\sin x)}{dx} = \sin x + x\cos x$ ✓

$\therefore x\cos x = \frac{d}{dx}(x\sin x) - \sin x$ ✓

$\int x\cos x \, dx = \int \frac{d}{dx}(x\sin x)dx - \int \sin x \, dx$ ✓

$= x\sin x + \cos x + C$ ✓

4. [2, 1, 3 marks]

(a) Find the equation of the plane through A (2, -3, 4) which has a normal vector of ⟨-1, 5, 3⟩.

$\underset{\sim}{r} \cdot \underset{\sim}{n} = \underset{\sim}{a} \cdot \underset{\sim}{n}$

$\underset{\sim}{r} \cdot \langle -1, 5, 3 \rangle = \langle 2, -3, 4 \rangle \cdot \langle -1, 5, 3 \rangle$ ✓

$= -2 - 15 + 12$

$= -5$

$\therefore -x + 5y + 3z = -5$ ✓

(b) Find the equation of the line through ⟨16, -17, -8⟩ and parallel to ⟨-2, 3, 1⟩.

$\underset{\sim}{r} = \langle 16, -17, -8 \rangle + \lambda \langle -2, 3, 1 \rangle$ ✓

$= \langle 16 - 2\lambda, -17 + 3\lambda, -8 + \lambda \rangle$

(c) Hence, find where the above line and plane intersect.

$\underset{\sim}{r} = \langle 16 - 2\lambda, -17 + 3\lambda, -8 + \lambda \rangle$

$\underset{\sim}{r} \cdot \langle -1, 5, 3 \rangle = -5$

$\langle 16 - 2\lambda, -17 + 3\lambda, -8 + \lambda \rangle \cdot \langle -1, 5, 3 \rangle = -5$

$-16 + 2\lambda - 85 + 15\lambda - 24 + 3\lambda = -5$ ✓

$20\lambda = 120$

$\lambda = 6.$ ✓

$\underset{\sim}{r} = \langle 16 - 12, -17 + 18, -8 + 6 \rangle$

$= \langle 4, 1, -2 \rangle$ ✓

$(4, 1, -2)$

5. [6 marks]

Show how to find the derivative of $\cos 2x$ from first principles if it is known that $\lim_{h \to 0} \frac{\cos 2h - 1}{h} = 0$.

$$\frac{d(\cos 2x)}{dx} = \lim_{h \to 0} \frac{\cos 2(x+h) - \cos 2x}{h}$$

$$= \lim_{h \to 0} \frac{\cos 2x \cos 2h - \sin 2x \sin 2h - \cos 2x}{h}$$

$$= \lim_{h \to 0} \frac{\cos 2x(\cos 2h - 1)}{h} - \lim_{h \to 0} \frac{\sin 2x \sin 2h}{h}$$

$$= \cos 2x \lim_{h \to 0} \frac{\cos 2h - 1}{h} - 2\sin 2x \lim_{h \to 0} \frac{\sin 2h}{2h}$$

$$= \cos 2x \cdot 0 - 2\sin 2x \cdot 1$$

$$= -2\sin 2x$$

6. [7 marks]

Prove that the square of every whole number has a remainder of either 1 or 0 when divided by 3.

Every whole no is $3n + k$, For n whole & $k = 0, 1, 2$

$k = 0$ $\quad (3n)^2 = 9n^2$

$= 3(3n^2)$

$\therefore$ no remainder when divided by 3.

$k = 1$ $\quad (3n+1)^2 = 9n^2 + 6n + 1$

$= 3(3n^2 + 2) + 1$

$\therefore$ remainder 1 when divided by 3

$k = 2$ $\quad (3n+2)^2 = 9n^2 + 12n + 4$

$= 9n^2 + 12n + 3 + 1$

$= 3(3n^2 + 4n + 1) + 1$

$\therefore$ remainder 1 when divided by 3

$\therefore$ every whole number has remainder 0 or 1 when divided by 3.

7. [3,7 marks]

Find the following:

(a) $\int \frac{3x}{\sqrt{2-3x^2}}\,dx$ without letting $u = 2 - 3x^2$.

$= \int 3x(2-3x^2)^{-1/2}\,dx$ ✓

$= \frac{3x(2-3x^2)^{1/2}}{\frac{1}{2}\cdot -6x} + C$ ✓

$= -(2-3x^2)^{1/2} + C$

$= -\sqrt{2-3x^2} + C$ ✓

(b) $\int_0^{\sqrt{3}} x\sqrt{4-x^2}$ by letting $x = 2\sin\theta$.

$x = 2\sin\theta$ ✓ $\qquad \sin\theta = \frac{x}{2}$

$\frac{dx}{d\theta} = 2\cos\theta \Rightarrow dx = 2\cos\theta\, d\theta \qquad x = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$ ✓

$x = 0 \Rightarrow \theta = 0$

$\int_0^{\sqrt{3}} x\sqrt{4-x^2}\,dx = \int_{\theta=0}^{\theta=\frac{\pi}{3}} 2\sin\theta . \sqrt{4-4\sin^2\theta} . 2\cos\theta\, d\theta$ ✓

$= \int_{\theta=0}^{\theta=\frac{\pi}{3}} 2\sin\theta \sqrt{4(1-\sin^2\theta)} . 2\cos\theta\, d\theta$

$= \int_{\theta=0}^{\theta=\frac{\pi}{3}} 2\sin\theta \sqrt{4\cos^2\theta} . 2\cos\theta\, d\theta$

$= \int_{\theta=0}^{\theta=\frac{\pi}{3}} 2\sin\theta\, 2\cos\theta . 2\cos\theta\, d\theta$ ✓

$= \int_{\theta=0}^{\theta=\frac{\pi}{3}} 8\sin\theta\cos^2\theta\, d\theta$ ✓

$= \left[-\frac{8}{3}\cos^3\theta\right]_{\theta=0}^{\theta=\frac{\pi}{3}}$ ✓

$= -\frac{8}{3}\cos^3\left(\frac{\pi}{3}\right) - \frac{-8}{3}\cos^3 0$

$= -\frac{8}{3}\left(\frac{1}{2}\right)^3 + \frac{8}{3}(1)^3$

$= \frac{-8}{3}\cdot\frac{1}{8} + \frac{8}{3}$

$= \frac{-1}{3} + \frac{8}{3}$

$= \frac{7}{3}$ ✓

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Sem 1 Examination 2010
Question/answer booklet

MATHEMATICS: SPECIALIST 3CDMAS

Section Two (calculator-assumed)

Name: ______________________________

Teacher: ______________________________

Time allowed for this section

Section Two

Reading time before commencing work:	10 minutes
Working time for paper:	100 minutes

Material required/recommended for this paper

To be provided by the supervisor

Question/answer booklet for Section Two and a formula sheet (from Curriculum Council) which was provided with Section One.

To be provided by the candidate

Section Two:

Standard items:	*pens, pencils, pencil sharpener, highlighter, eraser, ruler*
Special items:	drawing instruments, templates, and up to three calculators, (CAS, graphic or scientific), which satisfy the conditions set by the Curriculum Council for this course. Up to two (2) A4 pages of notes (handwritten, photocopied or typed on both sides) that may be either personally or commercially produced. *It must not be folded, have anything stuck to it or have correction fluid/tape on it.*

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MARK ALLOCATION AND RECORDS:

Section	Question	Marks	Awarded
TWO	8	3	
	9	3	
	10	4	
	11	4	
	12	4	
	13	4	
	14	5	
	15	5	
	16	5	
	17	6	
	18	8	
	19	10	
	20	18	
	Penalties	- 1/2/3	
	TWO	79	

Penalties	
Rounding (-1)	
Units (-1)	
Notation (-1)	

Section Two: Calculator – 79 marks

This section has **thirteen (13)** questions. Attempt **all** questions.

Working time: **100 minutes**

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8. [3 marks]

Sketch the set of complex number z which satisfy the following condition

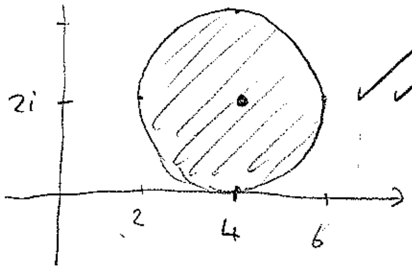
$$|z - 4 - 2i| \le |\sqrt{2} + i\sqrt{2}|.$$

$|z-(4+2i)| \le \sqrt{2+2}$ ✓

≤ 2

$\therefore$ circle centre $(4+2i)$

rad $= 2$



✓✓

9. [3 marks]

Use the ordinary rules of algebra to solve $\frac{z-i}{z+i} = 1 + i$ for z.
(i.e. don't substitute $z = x + yi$)

$z - i = (z+i)(1+i)$

$\cancel{z} - i = \cancel{z} + iz + i - 1$ ✓

$iz = 1 - 2i$

$z = \frac{1-2i}{i} \times \frac{i}{i}$ ✓

$= \frac{i+2}{-1}$

$= -2 - i$ ✓

10. [4 marks]

Find the derivative of $y = x - x^3$ from first principles.

$\frac{dy}{dx} = \lim_{h\to 0} \left(\frac{x+h-(x+h)^3 - (x-x^3)}{h} \right)$ ✓

$= \lim_{h\to 0} \left(\frac{x+h-x^3-3x^2h-3xh^2-h^3-x+x^3}{h} \right)$ ✓

$= \lim_{h\to 0} \left(\frac{h-3x^2h-3xh^2-h^3}{h} \right)$

$= \lim_{h\to 0} \left(\frac{h(1-3x^2-3xh-h^2)}{h} \right)$ ✓

$= \lim_{h\to 0} (1-3x^2-3xh-h^2)$

$= 1-3x^2$ ✓

11. [4 marks]

Show that $|a+z|^2 = a^2 + a(z+\overline{z}) + |z|^2$ for real a.

Let $z = x + iy$

LHS $= |a + x + iy|^2$

$= (a+x)^2 + y^2$

RHS $= a^2 + a(x+iy + x-iy) + |x+iy|^2$

$= a^2 + a(2x) + x^2 + y^2$

$= a^2 + 2ax + x^2 + y^2$

$= (a+x)^2 + y^2$

$=$ LHS

12. [4 marks]

Find the exact distance between A (5, 140°) and B (8, -160°).

$d^2 = 5^2 + 8^2 - 2(5)(8)\cos(300°)$

$= 49$

$\therefore d = 7$

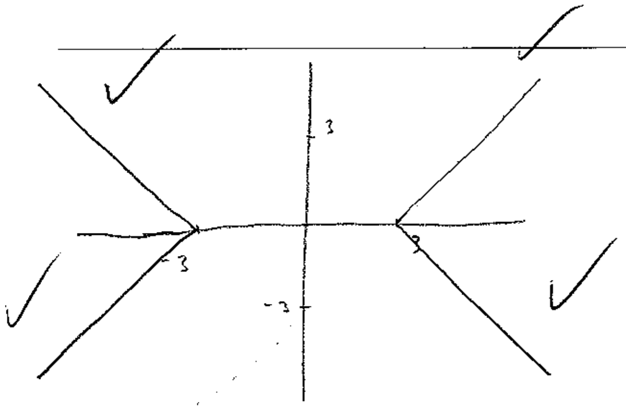
13. [4 marks]

Sketch the graph of $|x| - |y| = 3$.

$|y| = |x| - 3$

For $x > 0$ $\quad |y| = x - 3$

$x < 0$ $\quad |y| = -x - 3$



14. [5 marks]

Use de Moivre's rule to find the exact value of $(1 + i)^5 - (1 - i)^5$.

$= \left(\sqrt{2}\,\mathrm{cis}\left(\frac{\pi}{4}\right)\right)^5 - \left(\sqrt{2}\,\mathrm{cis}\left(\frac{-\pi}{4}\right)\right)^5$

$= (\sqrt{2})^5\,\mathrm{cis}\left(\frac{5\pi}{4}\right) - (\sqrt{2})^5\,\mathrm{cis}\left(\frac{-5\pi}{4}\right)$

$= 4\sqrt{2}\left(\mathrm{cis}\,\frac{5\pi}{4} - \mathrm{cis}\left(\frac{-5\pi}{4}\right)\right)$

$= 4\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} - \cos\left(\frac{-5\pi}{4}\right) - i\sin\left(\frac{-5\pi}{4}\right)\right)$

$= 4\sqrt{2}\left(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)$

$= 4\sqrt{2}\left(-2i\frac{\sqrt{2}}{2}\right)$

$= 4\sqrt{2}\,(-i\sqrt{2})$

$= -8i$

15. [5 marks]

Find $\int_0^{\frac{\pi}{3}} \cos^2\theta \sin^3\theta \, d\theta$ exactly by letting $u = \cos\theta$.

$\frac{du}{d\theta} = -\sin\theta \Rightarrow \sin\theta\, d\theta = -du$

$\int_{\theta=0}^{\pi/3} \cos^2\theta \sin^3\theta \, d\theta$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\cos 0 = 1.$

$= \int_{\theta=0}^{\pi/3} \cos^2\theta \sin^2\theta \sin\theta \, d\theta$

$= \int_{\theta=0}^{\pi/3} \cos^2\theta (1 - \cos^2\theta) \sin\theta \, d\theta$

$= \int_{u=1}^{u=1/2} u^2(1 - u^2) \cdot -du$

$= -\int_1^{1/2} u^2 - u^4 \, du = -\left[\frac{u^3}{3} - \frac{u^5}{5}\right]_1^{1/2}$

$= -\left(\frac{1}{24} - \frac{1}{160}\right) - -\left(\frac{1}{3} - \frac{1}{5}\right)$

$= \frac{47}{480}$

16. [5 marks]

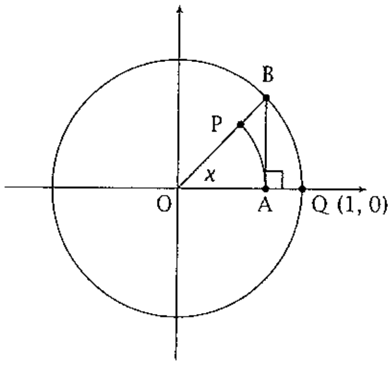
Consider the unit circle shown.

Use the fact that

length of arc AP < length of line AB < length of arc QB

to establish that
$x \cos x < \sin x < x$

and hence that $\lim_{x \to 0} \frac{\sin x}{x} = 1$.



$OA = \cos x$

$\therefore$ length arc $AP = x\cos x$

$\sin x = AB.$

length arc $QB = 1.x$

$\therefore x\cos x < \sin x < x$

$\cos x < \frac{\sin x}{x} < 1$

As $x \to 0 \quad \cos x \to 1$

$\therefore \lim_{x \to 0} \frac{\sin x}{x} = 1$

17. [6 marks]

If $z = r \operatorname{cis} \theta$, where r is constant and θ is variable, prove that
$\operatorname{Re} \int z \, d\theta = \int \operatorname{Re} z \, d\theta$.

LHS $= \operatorname{Re}\{\int r\cos\theta + ri\sin\theta \, d\theta\}$ ✓

$= \operatorname{Re}\{r\sin\theta - ir\cos\theta + c\}$ ✓

$= r\sin\theta + c$ ✓

RHS $= \int \operatorname{Re}(r\cos\theta + i\sin\theta)\, d\theta$ ✓

$= \int r\cos\theta \, d\theta$

$= r\sin\theta + c$ ✓

$=$ LHS ✓

18. [3,5 marks]

(a) Show how to solve $w^3 = -2 + 2i$ for one solution of w in exact cartesian form.

$w^3 = \sqrt{8} \operatorname{cis}\left(\frac{3\pi}{4}\right)$ ✓

$w = \left(2^{3/2} \operatorname{cis}\left(\frac{3\pi}{4}\right)\right)^{\frac{1}{3}}$

$= 2^{1/2} \operatorname{cis}\left(\frac{3\pi}{4} \cdot \frac{1}{3}\right)$

$= 2^{1/2} \operatorname{cis}\left(\frac{\pi}{4}\right)$ ✓

$= \sqrt{2}\cos\frac{\pi}{4} + i\sqrt{2}\sin\frac{\pi}{4}$

$= \sqrt{2} \cdot \frac{1}{\sqrt{2}} + i\sqrt{2}\frac{1}{\sqrt{2}} = 1 + i$ ✓

(b) Use your answer above to solve the equation below for one solution of z without substitution of $z = x + yi$.

$$(z - 1)^3 = -2(z + 1)^3 + 2i(z + 1)^3$$

$(z-1)^3 = (-2+2i)(z+1)^3$ ✓

$z - 1 = (-2+2i)^{1/3}(z+1)$ ✓

$= (1+i)(z+1)$ ✓

$= z + 1 + iz + i$ ✓

$z - z - iz = 1 + 1 + i$

$-iz = 2 + i$

$z = \frac{2+i}{-i} \times \frac{i}{i}$

$= i2 + 1$

$= -1 + i2$ ✓

19. [1, 2, 3, 2, 2 marks]

z and w are complex numbers such that $z = 1 - i$ and $w = 2e^{i\frac{\pi}{6}}$.
Find the following:

(a) $|z|$

$\sqrt{1+1} = \sqrt{2}$ ✓

(b) w in exact cartesian form

$2\operatorname{cis}\frac{\pi}{6} = 2\cos\frac{\pi}{6} + i\,2\sin\frac{\pi}{6}$ ✓

$= 2\frac{\sqrt{3}}{2} + i\,2\cdot\frac{1}{2}$

$= \sqrt{3} + i$ ✓

(c) $\bar{z}w$ in exact polar form

$z = \sqrt{2}\operatorname{cis}\left(\frac{-\pi}{4}\right)$ $\quad w = 2\operatorname{cis}\frac{\pi}{6}$

$\bar{z} = \sqrt{2}\operatorname{cis}\frac{\pi}{4}$ ✓

$\bar{z}w = 2\sqrt{2}\operatorname{cis}\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = 2\sqrt{2}\operatorname{cis}\left(\frac{5\pi}{12}\right)$ ✓ ✓

(d) $\frac{w}{z^2}$ in exact exponential form

$z = \sqrt{2}\,e^{-i\frac{\pi}{4}}$ $\quad \frac{w}{z^2} = \frac{2e^{i\frac{\pi}{6}}}{2e^{-i\frac{\pi}{2}}}$

$z^2 = 2\,e^{-i\frac{\pi}{2}}$ ✓

$= e^{i\frac{4\pi}{6}} = e^{i\frac{2\pi}{3}}$ ✓

(e) z^5 in exact polar form.

$z^5 = \left(\sqrt{2}\operatorname{cis}\frac{-\pi}{4}\right)^5$ ✓

$= 4\sqrt{2}\operatorname{cis}\frac{-5\pi}{4}$

$= 4\sqrt{2}\operatorname{cis}\left(\frac{3\pi}{4}\right)$ ✓

20. [7, 11 marks]

A jet travelling at ⟨-200, 150, 0.5⟩ km/hr passes through point (50, -20, 3.8) km at 2pm one day. At the same time a light plane is at (-238, 460, 4.52) km travelling at ⟨-80, -50, 0.2⟩ km/hr.

(a) If the aircraft continue as above, prove that the planes will collide and find the time and place of the collision.

$\underset{\sim}{r}_j(t) = \langle 50, -20, 3.8\rangle + t\langle -200, 150, 0.5\rangle$

$= \langle 50-200t, -20+150t, 3.8+0.5t\rangle$ ✓

$\underset{\sim}{r}_p(t) = \langle -238, 460, 4.52\rangle + t\langle -80, -50, 0.2\rangle$

$= \langle -238-80t, 460-50t, 4.52+0.2t\rangle$ ✓

Collide $\Rightarrow$ $\underset{\sim}{r}_j(t) = \underset{\sim}{r}_p(t)$ for some t.

i: $50-200t = -238-80t$

$288 = 120t$ ✓

$t = 2.4$ hrs

time = 4:24pm ✓

j: $-20+150t = 460-50t$

@ ⟨-430, 340, 5⟩km ✓

$200t = 480$ ✓

$t = 2.4$ hrs

k: $3.8+0.5t = 4.52+0.2t$

$0.3t = 0.72$ ✓

$t = 2.4$ hrs $\therefore$ collision

(b) After two hours of travelling the air traffic controller predicts that there could be a close encounter and orders the jet to change its velocity to ⟨-220, 130, 0.8⟩ km/hr. Find the closest distance that the two aircraft come to each other after this change.

At 2 hrs.

$\underset{\sim}{r}_j = \langle -350, 280, 4.8\rangle$ ✓ ✓

$\underset{\sim}{r}_p = \langle -398, 360, 4.92\rangle$ ✓ ✓

$\therefore$ after 2hr

$\underset{\sim}{r}_j(t) = \langle -350-220t, 280+130t, 4.8+0.8t\rangle$ ✓

$\underset{\sim}{r}_p(t) = \langle -398-80t, 360-50t, 4.92+0.2t\rangle$ ✓

$\underset{\sim}{r}_j(t) - \underset{\sim}{r}_p(t) = \langle 48-140t, -80+180t, -0.12+0.6t\rangle$ ✓✓

$d^2 = (48-140t)^2 + (-80+180t)^2 + (-0.12+0.6t)^2$ ✓✓

min distance = 11.23 km ✓